GRADE 4, 6 AND 8 STUDENTS' STRATEGIES IN AREA MEASUREMENT

Gillian C. Kidman
Centre for Mathematics and Science Education,
QUT, Brisbane, Australia
<g.kidman@qut.edu.au>

This paper investigates Grades 4, 6 and 8 students' confusion between area and perimeter. Students were given area judgment tasks involving rectangles and their responses analysed in terms of the operations used and the strategies exhibited. Students were found to use both additive (Perimeter) and multiplicative (Area) judgment rules, and 7 different strategies. Those using an additive judgment rule tended to rely on rulers or fingers for measuring length and to align the rectangles vertically, whilst those using a multiplicative judgment rule tended to use overlay and partitioning strategies.

It has been well documented in the literature (for example, Hart, 1981, Hirstein, 1981 and Kidman, 1997) that students often become confused between area and perimeter. This paper reports on a study investigating this confusion through the use of Information Integration Theory (IIT) (Anderson & Cuneo, 1978) which identifies groups of students who employ additive and multiplicative judgment rules when making comparisons between rectangular shapes.

Initial research into children's understanding of area and perimeter was conducted by Piaget and his associates (Piaget, Inhelder, & Szeminska, 1960). Since then, Piaget's description of the development of area concepts has been the subject of a wide variety of studies attempting to replicate, disprove, extend or explain his conclusions. These studies (for example, Hart, 1981, and Hirstein, 1981) covered the development of the major concepts of conservation and transitivity and dealt with the following four basic concerns:

(1) difficulties with validating the reality of individual cognitive operations and describing the stages of their development - this issue led to studies which replicated (with slight variation) some of Piaget's procedures and materials;

(2) difficulties in investigating the relationships between different cognitive skills -this second issue was addressed by researchers administering a series of different tasks to the same sample of children to establish a developmental hierarchy;

(3) misconceptions children have relating to area - information on this issue was obtained by analysing incorrect answers provided by children and adults; and

(4) difficulties with identifying the nature of the progression between stages of development - this was dealt with through training studies.

The research questions gradually changed over time. Piaget and his associates published their findings in 1960. During the next 15 years, most area research centred around replicating Piaget's work. After 1975, replicating studies were rarely seen, with the exception of Lu's work (1981, 1991) from China. Area research focused on investigating relationships between different cognitive skills, and on large scale assessments of the performance of children on a variety of area tasks (Bell, Hughes, & Rogers, 1975; Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Foxman, Ruddock, Badger,& Martini, 1982; Hart, 1981). Studies concerned with student misconceptions emerged from this availability of data related to children's understanding.

Using the data from the mathematics section of the second National Assessment of Educational Performance, Hirstein (1981) studied the causes of errors and found that the difficulties shown by the students seemed to result from misconceptions about area rather than from computational weaknesses. Students were often unable to select the appropriate procedure when the link between the question and the computation was not explicitly made. This finding was supported by Foxman, Ruddock, Badger and Martini (1982) who found that students were unclear in the choice of correct strategy in a practical test. However, the particular problem students have with area is confusion with perimeter (Foxman *et al.*, 1982; Gholam, 1994; Hirstein, 1981; Hirstein, Lamb, & Osborne 1978; Kidman, 1997; Nitabach & Lehrer, 1996). Students found the process of obtaining a shape's measurements difficult, especially with regard to which dimensions to consider and how to count the units along the selected dimensions. A number of studies have also found confusion with reading and writing the Standard International area measurement units (Baturo & Nason, 1996; Foxman *et al.*, 1981).

This paper reports on a study investigating the confusion between area and perimeter. This study determined Grades 4, 6 and 8 students' area judgment rules by using Information Integration Theory (IIT) (Anderson and Cuneo, 1978). The aims of the study were: (1) to identify groups of students who have additive judgment rules, and groups of students who have multiplicative judgment rules; (2) to identify the strategies for judging area used by these students; and (3) to relate strategy usage to judgment rule.

METHOD

The study used a multi-method design where the quantitative methodology of IIT was combined with the qualitative methodology of a semi-structured clinical interview. Thirty six students, from the same private college, were interviewed, twelve students from each of the three grade levels, 4, 6 and 8. The students represented a range of mathematical abilities, one third each of below average, average, and above average. (A comprehensive outline of the methodology of the study, including how IIT determines area judgement rules, is provided in Kidman, 1997, and Kidman & Cooper, 1996.)

In the interview, students were provided with 16 rectangular wooden pieces painted to represent chocolate bars. Four widths (3, 6, 9 and 12 cm) factorially combined with the same heights made the 16 stimuli pieces used in the experiment. As the pieces were presented to them, the students were asked to rate the area of the wooden pieces by stating how happy or sad they would be to receive that amount of chocolate in relation to two end anchor pieces. To obtain a rating of the their judgments, the student was provided with a 19 point scale with two end points. The student responses for these 16 stimuli were plotted (using the methodology of Anderson and Cuneo (1978)) against the length of the rectangles.

If the resulting plot was parallel (a collection of parallel lines or curves), this reflected a perception of area where doubling the lengths of both the sides of the rectangle was seen as doubling the area. Therefore, students with parallel plots were considered to have additive area judgments, that is, the students perceived the area of a rectangle in terms of the *sum* of its dimensions. This perception was considered a confusion between area and perimeter for rectangles (Kidman, 1997), and the area integration rule it represented was denoted as *Area = Height + Width.*

If the plot was fan shaped (expanding lines or curves), this reflected a perception where doubling the sides more than doubles the area of the rectangle. Therefore, students with fan-shaped plots were considered to have multiplicative area judgements, that is, the students tended to see the area of a rectangle in terms of the *product* of its dimensions. The area judgement rule represented by this perception was denoted as *Area = Height x Width.*

The students' actions as they determined their rating for each wooden piece were documented, and the students were also asked to describe their thinking when they were determining their rating.

RESULTS AND DISCUSSION

The actions and statements of the students revealed that all students understood what they had to do with the ratings and saw the rating as determining area (see Kidman, 1997; Kidman & Cooper, 1996.) For all but one of the interviews, the resulting plots were obviously additive or multiplicative, with clear parallel or fan-shaped plots. The groups of students who applied additive and multiplicative area judgment rules, the seven strategies used for judging area, and the strategy usage in relation to area judgment rules, are now provided.

Judgment rules

Both the additive and multiplicative judgment rules were evident in the students' responses. 19 of the 36 students employed an additive judgment rule, while 16 employed the multiplicative judgment rule. The occurrence of the additive rule was consistent across the grades indicating a strong misconception for the area of a rectangle being related to the sum of the rectangles' dimensions.

It was not possible to determine a judgment rule for one Grade 4 student (Ben). This particular plot had four intersecting locations and no obvious parallel curves or diverging lines. Plots with lines crossing indicate a very poor conception of area. This student judged a rectangle with smaller dimensions as having a larger area than a rectangle with larger dimensions.

Strategies

There were seven strategies used by the students. Most of the students displaying an additive judgment rule used more than one strategy, whereas most of the students displaying a miltiplicative judgment rule employed only one strategy.

1. *Ruler Request strategy (RR).* When asked if there were any unclear aspects of the experiment that were of concern, most students indicated that they fully understood the requirements and were ready to begin the actual experiment. However, the facial expressions of 8 students, 5 from Grade 4 and 3 from Grade 6, indicated that they were not happy to proceed to begin. These students all requested the use of a ruler when asked what the problem was: "Can I use a ruler, please?" (Jemma, Tom and Sam); "I need a ruler to do this" (Martin, Ben and Jack); "This can't be done properly without a ruler" (Jodie and Angie). Jack (Grade 4) explained that he needed to "measure the chocolate pieces to see if one was bigger or not" . He wanted to measure the longest sides of each piece. Jodie (Grade 6) insisted that "we always use rulers to measure. You see, without one you can't measure something". Ben (Grade 4) needed "a small ruler to do this". Ben explained that "a big ruler is the one what the teacher used on the board, but we use small ones about this long ... [Ben indicated a length of approximately 25 cm]. They change the size [of the stick] to fit our books".

With the exception of Peter (Grade 6) and Ben (Grade 4), all students requesting the use of a ruler employed an additive judgment rule. Ben's plot was not able to be classified as either additive or multiplicative, while Peter exhibited a multiplicative judgment rule.

2. *Index Finger strategy (IF).* Eight students, four students from Grade 4 and two each from Grades 6 and 8, used their index finger to repeatedly measure one side of the rectangular test piece. Initially the student placed an index finger adjacent and parallel to one side of the rectangular piece. This is then repeated by moving the finger along to the next adjacent parallel position. The student repeated this procedure along the edge of the rectangular piece to the opposite side. A value is then assigned to the piece and a rating made. It was a strategy used only by students employing the additive judgment rule. It seems that, in the absence of a ruler, some of the children who perceive the area of a rectangle will

double if you double the length of both sides will improvise by using the index finger. The length of the finger is overlaid on the test piece and the number of times this occurs is directly related to the rating response.

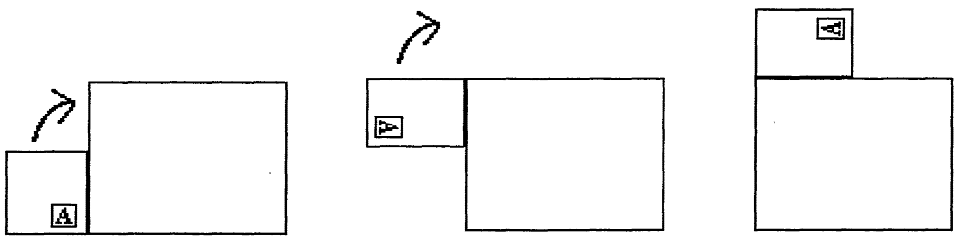
3. *Vertical Alignment strategy (VA).* Each of the wooden pieces were presented to the students in a uniform manner, but some pieces were presented in a horizontal alignment (lying flat ▭), while others had a vertical alignment (standing tall ▯). Six students from Grade 4, another six from Grade 6, and four from Grade 8, rotated the rectangular test pieces when presented with a horizontal alignment. Students using this strategy mostly employed an additive judgment rule (80% of the additive students favoured a vertical alignment of the rectangular test pieces). The strategy was particularly prevalent for the larger pieces.

Rhea (Grade 6) explained that the alignment of the test pieces was very important: "You gave me it [the test piece] like this ... [horizontal alignment] ... kind of lying down, but chocolate bars are stacked like this [she realigns the test piece to a vertical orientation] on shop shelves, so you need them put here in the same way ... like the way the wrapper would go". However, most students did not know why they repeatedly realigned the test pieces, for example, Sam (Grade 6) simply stated "It just feels better", while Steven (Grade 8) claimed "It makes things easier".

4. *Test piece Rotation strategy (TR).* One student from Grade 4 (additive judgment rule), and two students from Grade 8 (one additive and one multiplicative judgment rule) employed a strategy where they measured the outside edge of the rectangular piece (rotating the piece so each edge can be compared) against the end anchors (see Figure 1).

Sue explained: "You get the piece and match up the side with the ... um ... this one [selected the large end anchor] and turn the piece all around it edge. That gives you the count of its size. If the count is big, you rate it small". Sue has some understanding of inverse proportion, but confused perimeter (the distance around each shape) and area (the internal space of the shape).

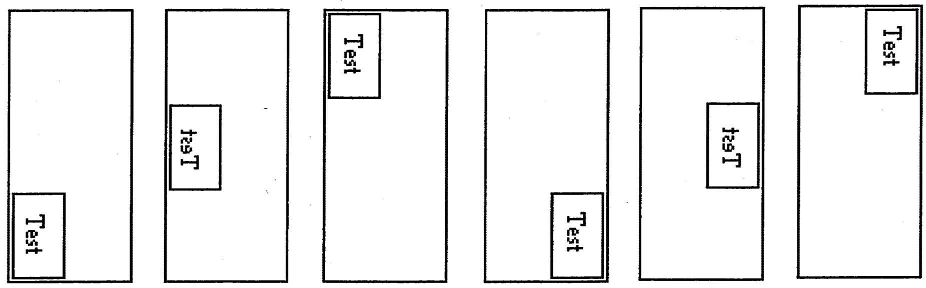
Figure 1
Test Piece Rotated Around the End Anchor



5. *Test piece overlay on end Anchor strategy (TA).* Three Grade 4, two Grade 4 and two Grade 6 students used a strategy of overlaying the test pieces onto the large end anchor (see Figure 2). The test piece was moved over the surface of the end anchor in a series of flip and slide transformations. The students counted the number of times the test pieces were moved to cover the end anchor. Only students employing a multiplicative judgment rule displayed this strategy.

While this strategy shows an understanding of area as being coverage of space, it also reveals a possible misconception for the students. It had never occurred to both Matt and Mick (Grade 8 students) that the shape of the overlay piece was important except that as Matt pointed out "circles are no good, they leave gaps", while Mick knew that "you need shapes with points that fit together". John (Grade 4) thought that rectangles could only be used to tile rectangles, squares to tile squares, and so on.

Figure 2
Test Piece Overlay Strategy



The students using the test piece overlay strategy were unable to articulate the process by which they converted an overlay quantity into a rating response. As Peter (Grade 4) stated, "... just pointed to where it seemed right".

6. *End anchor overlaid onto Test piece strategy (AT).* The small end anchor was a square. Two students from Grade 6 and one from Grade 8 (all multiplicative judgment rule users) employed a strategy where they overlaid the small end anchor onto the test pieces. Jenny (Grade 6) stated "... counted how many squares, see these little squares [indicating the small end anchor], I want to see how many of them cover this bit [holding up the test piece]". Jay (Grade 8) explained "it has to be the small square that you use because it doesn't change each time like the bigger ones do. You have to have the same measuring piece all the time, and this [indicating the small end anchor] is good because it is a square. Well I think it looks like one ... um ... the sides look the same".

The students counted how many of the small end anchors fitted onto a test piece, demonstrated this using a series of randomised flip and slide transformations and then decided on the rating. Again, the students using this strategy were unable to articulate the process they used in converting an overlay quantity into a rating response. Anne (Grade 6) said she "just remembered from the piece before, and then made it bigger or smaller".

7. *Partitioning strategy(P).* One student from Grade 4, two from Grade 6 and two from Grade 8 (all using a multiplicative judgment rule) displayed partitioning strategies. These students were familiar with the method of partitioning both the length and width of a rectangular shape, and integrating these values using multiplicative reasoning.

One of the students, Kim (Grade 4) was unable to visualise such partitions. Instead she made imaginary marks along the two salient dimensions with her finger (see Figure 3). Initially Kim partitioned the width of the test piece into what appeared to be 1 cm lengths, maintaining a mental count of the partitions. Upon completion of the width, her attention focused on the length of the test piece and she repeated the partitioning process, again with 1 cm partitions, maintaining a mental count as she progressed. Kim explained she "... got the width number and times it by the length number. That gives the number of squares fitting onto the shape to tell how big it is". Kim had helped her grandfather lay paving tiles, and he had explained this to her.

As the four students from Grades 6 and 8 all explained, they imagined the chocolate pieces already divided into squares "like proper chocolate is supposed to be" (Wade, Grade 6), counted the squares on two dimensions, and multiplied "... them together, like you do for area sums" (Phillip, Grade 8).

Figure 2
Kim's Imaginary Partitioning Marks

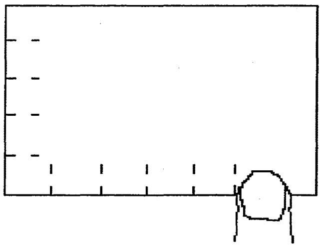


Table 1
Area Judgement Rules and Strategies

Student	Rule	RR	IF	VA	TR	TA	AT	P
Jemma 4	A	■	■	■				
Cassie 4	A		■	■				
Stacey 4	A		■					
Martin 4	A	■		■	■			
Tom 4	A	■		■				
Tony 4	A		■	■				
Jack 4	A	■		■				
Dianne 6	A			■				
Rhea 6	A		■	■				
Jodie 6	A	■		■				
Angie 6	A	■		■				
Sean 6	A		■	■				
Sam 6	A	■		■				
Carol 8	A		■					
Brooke 8	A		■					
Elle 8	A			■				
Kelly 8	A			■				
Sue 8	A			■	■			
Steven 8	A			■				
Ben 4	?	■						
Total	***19A 1?***	***8***	***8***	***16***	***2***	***0***	***0***	***0***

Alice 4	M					■		
Cathy 4	M					■		
Kim 4	M							■
John 4	M					■		
Jenny 6	M						■	
Anne 6	M						■	
Josè 6	M					■		
Wade 6	M							■
Peter 6	M	■				■		
Mark 6	M							■
Helen 8	M				■			
Phillip 8	M							■
Mick 8	M					■		
Matt 8	M					■		
Jay 8	M						■	
Kevin 8	M							■
Total	***16***	***1***	***0***	***0***	***1***	***7***	***3***	***5***

KEY Rule: A = Additive judgment rule; M = Multiplicative judgment rule; ? = Unclear - intersecting lines

Strategy: RR = Requested Ruler; IF = Index Finger; VA = Vertical Alignment; TR = Test piece Rotated; TA = Test piece overlay on end Anchor; AT = end Anchor overlay on Test; P = Partitioning

Link between strategy usage and judgment rules

Table 1 shows the seven strategies exhibited by the students in the study, for each of the two area judgment rules. It is clear that there is a strong link between judgment rule and the strategies employed. Students with a misconception of area relating to the additive properties of perimeter tended to require a ruler or use an index finger for measurement, and prefered a vertical alignment of the rectangle. Students using a multiplicative judgment rule tended to use overlaying strategies, involving both flips and slides of either the test piece or the small end anchor, and the more sophisticated partitioning strategy. The strategy of test rotation did not appear to be linked with either judgment rule.

CONCLUSION

The differences between the grades were not as apparent as expected (as can be seen in Table 1). The misconception of area of rectangles being dependent on the sum of the rectangles' dimensions is fairly constant across the grades. As a group, the Grade 8 students do not seem to have progressed beyond the Grade 6 level and not much beyond the Grade 4 level.

This experiment has, therefore, supported the claims of Anderson and Cuneo (1978) that students use both additive and multiplicative judgment rules. The findings of Hirstein (1981) and Hirstein *et al.* (1978) have also been supported in that there is confusion between area and perimeter. Around 50% of students from all grades exhibited judgments that showed they were using the perimeter rule to determine area. Those students using an additive judgment rule tend to want something with which to measure (ruler or index finger) and prefer a vertical alignment. Students using a multiplicative judgment rule tended to use overlay strategies as well as partitioning.

The findings of the study suggests that these students, particularly those in Grades 6 and 8, may not have had sufficient opportunity to explore practically the spatial foundations of area and perimeter and the relationships between them. The author suggests structured classroom activities of a practical nature (for example, the use of geoboards and dot paper) be more widely used to develop the notions of area and perimeter, and a delay in the presentation of area and perimeter formulae.

REFERENCES

Anderson, N.H., & Cuneo, D.O. (1978). The height + width rule in children's judgements of quantity. *Journal of Experimental Psychology: General, 107*(4), 335-378.

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics. 31*(3), 235-268.

Bell, D., Hughes, E. & Rogers, J. (1975). *Area, Weight and Volume*. London: Thomas Nelson & Sons.

Carpenter, T.P., Corbitt, M.K., Kepner, H.S., Lindquist, M.M., & Reys, R.E. (1981). *Results of the Second Mathematics Assessment Of The National Assessment Of Educational Progress*. Reston, Virginia: National Council of Teachers of Mathematics.

Foxman, D.D., Ruddock, G.J., Badger, M.E., & Martini, R.M. (1982). *Mathematical development: Primary Survey Report No. 3*. London: Her Majesty's Stationary Office.

Gholam, G.K. (1994). Children's strategies in measurement. *Mathematics in School*, Sept., 43-46.

Goodnow, J.J. (1968). A test of milieu effects with some of Piaget's tasks. *Psychological Monographs, 76* (36), Whole No. 555.

Hart, K. (1981). Measurement. In K.M. Hart (Ed.), *Children's Understanding of Mathematics:11-16* (pp.9-22). London: John Murray.

Hirstein, J.J. (1981). The second national assessment in mathematics: Area and volume. *Mathematics Teacher, 74*(9), 704-707.

Hirstein, J.J., Lamb, C.E., & Osborne, A. (1978). Student misconceptions about area measure. *Arithmetic Teacher, 25*(6), 10-16.

Kidman, G. (1997). Area integration rules. In F. Biddulph & K. Carr (Eds.) *Proceedings of the 20th Annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 271 - 277). New Zealand: Mathematics Education Research Group of Australasia.

Kidman, G. & Cooper, T.J. (1996). Children's perceptual judgement of area. In P.C. Clarkson (Ed.) *Proceedings of the 19th Annual Conference of the Mathematics Education Research Group of Australiasia* (pp. 330-336). Melbourne: Mathematics Education Research Group of Australiasia.

Lu, J. (1981). A researchon the logical reasoning ability of 4-9 year old children: Verifications of some of Piaget's experiments. *Acta Psychologica Sinica*, *13*(1), 30-34.

Lu, J. (1991). A research on children's understanding of geometric figures: A discussion of Piaget's cognitive theory of "From topology to Euclidean geometry". *Psychological Science China*, 3, 6-11.

Nitabach, E., & Lehrer, R. (1996). Developing spatial sense through area measurement. *Teaching Children Mathematics*, April, 473-476.

Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The Child's Conception Of Geometry.* New York: Basic Books.

Endnote

1. The assistance of Assoc. Prof. T.J. Cooper is appreciated in the preparation of this paper.